HOW TO CALCULATE

SCHILLINGER RHYTHMIC RESULTANTS

(AN ALTERNATIVE TO THE GRAPHING TECHNIQUE)

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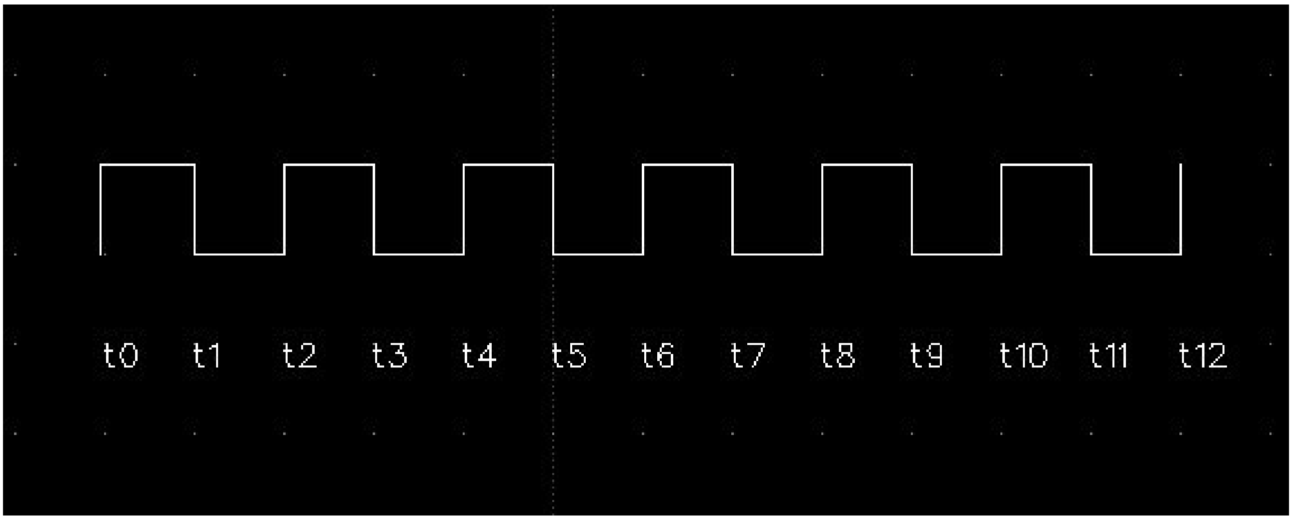
If you have ever been in a position of having a few moments to spare where you decide to try out a few rhythmic resultants I have often found graph paper not to hand. Without a grid to work to freehand sketching is difficult (though of course not impossible). Here is an alternative method to derive rhythmic resultants by calculation. It involves a mathematical theory that was not available to Schillinger and that is the modern theory of sets.

To begin with we note that a pulse (the unity beat) in graph form has two different horizontal levels connected together by vertical lines that occur at regular time intervals t_0, t_1, t_2 …..... such that the difference between two consecutive time intervals are equidistant.



This pulse train can be succinctly written as (0,1,2,3,4,5,6,7,8,9,10,11,12), where the difference between successive time intervals is unity.

Similarly a major generator of 3 and a minor generator of 2 for instance could be written as (0,3,6,9,12,15.........) and (0,2,4,6,8,10........).

To begin to calculate $r_{3:2}$ we must first determine the product of a and b which in this case is

$$ab = 3 \times 2 = 6$$

Now we can write a = (0,3,6) and b = (0,2,4,6) where we have terminated our listings of the pulse trains 3 and 2 when we reach there product 6. In the language of mathematical sets we now have two sets a = (0,3,6) and b = (0,2,4,6)

To calculate $r_{3:2}$ we simply form the union of the two sets a and b. This means we form the new set a U b (where U means union) by forming a new set that lists all of the elements of both sets a and b without repetition of elements and ordering the elements in increasing value. Thus in this example

$$a \cup b = 3 \cup 2 = (0,3,6) \cup (0,2,4,6) = (0,2,3,4,6)$$

We now remember that (0,2,3,4,6) means a graph of pulses whose vertical lines occur at t_0, t_2, t_3, t_4, t_6 which is our resultant of 3:2 . Taking the difference of successive time intervals we get

$$(2 – 0) + (3 – 2) + (4 – 3) + (6 – 4) =$$

$$2 + 1 + 1 + 2$$

thus

$$r_{3:2} = 2 + 1 + 1 + 2$$

It only takes a little practice using this method before you find that it is quicker to calculate resultants than to graph them.

Here is another example. The resultant of 8:5. 8 x 5 = 40 so 40 is where we terminate each pulse train giving

a = (0,8,16,24,32,40) and b = (0,5,10,15,20,25,30,35,40)

thus

a U b = (0,5,8,10,15,16,20,24,25,30,32,35,40)

and now taking the difference between every two consecutive elements of a U b we get

$r_{8:5}$ = 5 + 3 + 2 + 5 + 1 + 4 + 4 + 1 + 5 +2 + 3 + 5

With a little extra technique resultants with fractioning may also be calculated using this method.

Instead of the product ab being the terminating element we must now use a^2. Further more there will be more than one occurrence of the b generator with each b set commencing on the successive elements from a and terminating on reaching the next coincident element of a. The union of the a set with all the b sets will show the timing of the resultant with fractioning of a:b

A couple of examples will make this clearer. First $r_{3:2}$

$a^2 = 3^2 = 9$

therefore

a = (0,3,6,9)

and

b_1 = (0,2,4,6)

b_1 starts on the first element of a and ends on 6 which is the next element in a that the 2 pulse finds agreement. The second b set must start on the second element of a in this case 3 hence

$$b_2 = (3,5,7,9)$$

in this example the terminating element of b_2 is 9 which is also the terminating element of a and so there are no more b sets. Otherwise we would have carried on to another b set starting on the third element of a terminating again on the next element of a in which a b element is equal. And so on until we reach the position as we have in this example where the last b set and the a sets terminating element is a^2. So in our current example we have the three sets

$$a = (0,3,6,9),\ b_1 = (0,2,4,6), \text{ and } b_2 = (3,5,7,9)$$

we now form the union of the three sets to get

$$a \cup b_1 \cup b_2 = (0,2,3,4,5,6,7,9)$$

therefore

$$r_{\underline{3:2}} = 2 + 1 + 1 + 1 + 1 + 2$$

One final example $r_{\underline{8:5}}$

$$a^2 = 8 \times 8 = 64$$

$$a = (0,8,16,24,32,40,48,56,64)$$

$$b_1 = (0,5,10,15,20,25,30,35,40)$$

$$b_2 = (8,13,18,23,28,33,38,43,48)$$

$$b_3 = (16,21,26,31,36,41,46,51,56)$$

$$b_4 = (24,29,34,39,44,49,54,59,64)$$

$a \cup b_1 \cup b_2 \cup b_3 \cup b_4 =$

(0,5,8,10,13,15,16,18,20,21,23,24,25,26,28,29,30,31,32,33,34,35,36,38,39, 40,41,43,44,46,48,49,51,54,56,59,64)

therefore $r_{\underline{8:5}} =$

5 + 3 + 2 + 3 + 2 + 1 + 2 + 2 + 1 + 2 + 3(1) + 2 + 8(1) + 2 + 3(1) + 2 + 1 +2 + 2 + 1 + 3 + 2 + 3 + 5

where 3(1) = 1 + 1 + 1 and 8(1) = 1 + 1 + 1+ 1 + 1+ 1 + 1+ 1

I rarely graph resultants these days as the above method is so much quicker and does not require graph paper. I offer this as an alternative which I am sure some of you reading this will find of use.